GAIN AND LOSS CHARACTERISTICS OF "AVERAGE VALUE" ASSET VALUATION METHODS

An essential element of any actuarial valuation for ERISA funding purposes is the method of determining the actuarial value of assets. Section 412(c) of the Internal Revenue Code states:

> "...the value of the plan's assets shall be determined on the basis of any reasonable actuarial method of valuation which takes into account fair market value and which is permitted under regulations prescribed by the Secretary."

Internal Revenue Service regulation §1.412(c)(2)-1 sets forth the applicable guidelines for determining reasonableness. To be considered reasonable under the regulations the method of valuation must:

(1) Be applied on a consistent basis.

(2) Reflect either fair market value or average value.

(3) Not be designed to produce values consistently above or below either fair market value or average value.

(4) Be within 20% of fair market value or within 15% of average value.

Average value is defined as the average of the current fair market value and adjusted values of assets over a stated period not to exceed the five most recent plan years (including the current year). This can be written mathematically as:

(1)
$$A_x = \frac{1}{n}\left(M_x + \sum_{t=1}^{n-1} V_t^{x-t} \right), \quad n \leq 5$$

When $x =$ Current valuation date

$A_x =$ Actuarial value of assets at x

$M_x =$ Fair market value of assets at x

$V_t^x =$ Adjusted value based on initial value at time x and adjusted for t years

$V_0^x = M_x$

Under the regulations, the adjusted value as of a date is equal to the previous year's adjusted value increased for contributions, interest and dividends, and decreased for benefits paid and expenses. Therefore the adjusted value does not reflect subsequent realized gains (losses) or unrealized gains (losses). This produces the following equation for adjusted values:

(2) $V_t^x = V_{t-1}^x + T_{t-1}$

Where $T_t = C_t + I_t - B_t - E_t$ representing contributions, investment income (interest and dividends) benefit payments and expenses, respectively during year beginning at time t.

Thus, market value increases in any year as follows:

(3) $M_t = M_{t-1} + T_{t-1} + G_{t-1}$

Where $G_t = R_t + U_t$ representing realized gains (losses) and unrealized gains (losses) during year t.

Since $V_o^x = M_x$:

(4) $V_t^x = M_x + \sum_{k=x}^{x+t-1} T_k$. Likewise,

(5) $M_{x+t} = M_x + \sum_{k=x}^{x+t-1} (T_k + G_k)$. By taking the difference between equation (4) and (5) we find that:

$$M_{x+t} - V_t^x = \sum_{k=x}^{x+t-1} (R_k + U_k)$$

In other words, the adjusted value at year x + t differs from the market value at year x + t by the total of realized and unrealized gains and losses during the t years of adjustment.

Applying these adjusted values to equation (1) yields:

(6) $A_x = M_{x-n+1} + \sum_{k=1}^{n-1} T_{x-k} + \frac{1}{n}\sum_{k=1}^{n-1} k\, G_{x-k},\ n \leq 5$

Therefore the average value differs from market value in that under the average value, gains and losses are amortized on a straight line basis over a period of n years.

Examining successive values of A_x reveals:

(7.1) $A_{x+1} - A_x = T_x + \frac{1}{n}\sum_{k=0}^{n-1} G_{x-k}$, or substituting for T_x,

(7.2) $= C_x - B_x - E_x + I_x + \frac{1}{n}\sum_{k=0}^{n-1} G_{x-k}$

The first two terms of equation 7.2 represent cash transactions during a year while the last two represent investment return. The expense term can be a cash transaction, a negative component of investment return or zero depending on plan practice and actuarial assumptions. For purposes of this paper, it will be assumed that expenses will be $0.

The actuarial asset gain (or loss) is determined during a plan year by comparing the actual investment return on the actuarial value of assets to the expected return on the plan assets expressed as follows:

(8) $AG_x = I_x + \frac{1}{n}\sum_{k=0}^{n-1} G_{x-k} - (i\,Ax + i_T)$

Where AG_x equals actuarial gain from investments in year x, i equals the assumed valuation rate, and i_T equals the expected return on transactions due the year x.

Using equation (8) the characteristics of gains (or losses) under the regulatory average value method can begin to be evaluated. Central to this are the relative magnitudes of I_x and $\frac{1}{n}\sum_{k=0}^{n-1} G_{x-k}$ compared to $(iA_x + i_t)$.

Please note that while interest and dividends are recognized immediately under this method, realized and unrealized gains (or losses) are recognized over a period of n years. Therefore, the return on an investment that is expected to appreciate is not assigned as much credibility as the return on an investment that generates its return through interest and dividends. To illustrate this point, consider two investments. This first investment is an 8% treasury bond purchased at par value with interest payments reinvested at 8%. The other investment is a treasury strip bond purchased at a discount to yield 8%. Assume that the actual market returns are 8% annually over the next 10 years and that all returns can be likewise reinvested to yield 8% annually. The market values and actuarial values using a five year average value of each of these investments would be as follows:

	8% Coupon		Treasury Strip	
Year	Market	Actuarial	Market	Actuarial
0	1,000	1,000	1,000	1,000
1	1,080	1,080	1,080	1,040
2	1,166	1,166	1,166	1,082
3	1,260	1,260	1,260	1,127
4	1,360	1,360	1,360	1,173
5	1,469	1,469	1,469	1,267
6	1,587	1,587	1,587	1,369
7	1,714	1,714	1,714	1,478
8	1,851	1,851	1,851	1,596
9	1,999	1,999	1,999	1,724
10	2,159	2,159	2,159	1,862

As can be seen, because the value of the treasury strip is derived from appreciation instead of interest income, its actuarial value is lower in years 1 through 10. Is this reasonable? To answer this question, the nature of pension investments must be examined.

The pension trustee as fiduciary is charged under ERISA to discharge his duties "with the care, skill, prudence and diligence ... that a prudent man ... would use ... by diversifying the investments ...." Implicit in those charges is that a trustee will normally invest in assets of various classes and that the investments should be expected to be productive.

Among the various classes of assets available, some emphasize income — such as high coupon bonds — and some emphasize appreciation — such as growth stocks or zero coupon bonds. A prudent trustee would not be willing to invest in a low income type of asset without an expectation of appreciation to compensate for the lower current income.

Therefore, in any investment, the prudent investor makes an assumption of growth commensurate with other available investments. Historically, investments have been expected to produce positive returns. Certain investments, by design, produce very little growth by income. Accordingly, an implicit assumption is being made that some, if not all, of the asset growth will occur due to appreciation. This implies that the true underlying values of many invested plan assets are assumed to increase on a long term basis.

However, as stated by Arthur Andersen in Pension Mathematics for Actuaries, the IRS average value method is valid only if the "real or mean value of each yearly share price is consistent for all years. If it "... fluctuates in any predictable manner, then the method is invalid because in such cases a linear average of it is not an unbiased estimate ...." In fact, the mean value share prices of productive investments are generally expected to fluctuate in a predictable manner (i.e., increase). Accordingly the IRS average value method is a biased estimate of the true value of assets.

Thus, assuming that asset appreciation is expected to occur, gains are deferred under the average value method. To obtain a greater understanding of what this implies, it is helpful to develop a theoretical basis on which to evaluate asset valuation methods. The basis will provide the characteristics that a desirable asset valuation method should have with the understanding that it is unlikely that any asset valuation method can fully satisfy all of the criteria. In fact, some tradeoffs should be expected to the extent that the criteria are conflicting.

First, the method should not be designed to produce values consistently above or below fair market value. This condition is identical to the third condition stated in IRS §1.412(c)(2)-1 except that the use of average value is removed. Thus, use of average value can be tested against the basic IRS rule.

Second, over a period of time, if valuation assumptions are realized (i.e., the actual return equals the assumed return), the method should produce asset values which converge on fair market value. In other words, if a fund consistently earns the assumed return, the asset valuation method should produce values which approximately equal fair market value.

Third, the asset valuation method should not influence investment decisions. For example, if the actuarial value of assets is equal to book value, assets that are sold get marked to market value. If actuarial gains and losses are manipulated by the selective sale of assets, the asset valuation method is influencing investment decisions. As another example, under the regulatory average value method, a high income asset may be viewed as preferable to a high appreciation asset because the returns would be more rapidly recognized in the valuation assets. Once again, in this situation the valuation method would be influencing investment decisions

Fourth, a valuation method in any given year should generally move the actuarial value towards the fair value of assets. This characteristic is similar to the second characteristic but focuses on the progression from one year to the next. The second characteristic is a measure of the long term ability of an asset smoothing method to produce reasonable results. For example, assume that at the start of that year x the actuarial value is 5% greater than the market value. During year x the market value increases 10%. It is desirable for a smoothing method to produce an actuarial value at the start of year x+1 that was greater than it was one year earlier.

Fifth, a valuation method should produce some predictability of results from year to year. In other words, to what extent can the actuarial value at x+1 be predicted based on the actuarial value at year x. If the difference between the actuarial value and market value is viewed as a reserve, this implies that the reserve should be utilized in some predictable manner. An additional element of predictability lies in the manner that the current year's performance affects the actuarial value.

Sixth, since one of the primary purposes of an actuarial asset valuation method is to smooth out gains and losses, it should be expected to cumulatively produce lower gains and losses than that produced without smoothing, i.e., using straight market value.

A final condition would ideally be that the method would reflect the underlying true value of the asset. Of course, this is the crux of the issue. The true value of an asset is never known until the asset is converted to cash. The market value provides a snapshot value but, as evidenced by daily market fluctuations, cannot be considered as the true value. Accordingly, while this condition of a smoothing method appear intuitively desirable, it is unattainable. Instead, the above six conditions provide the best surrogate as to the ability of a valuation method to reflect true value.

With these conditions in mind, a model asset fund can be constructed based on contributions, benefit payments, income (e.g., interest and dividends) and appreciation, both realized and unrealized. Again, for simplicity sake, expenses will be ignored. All transactions will occur at year end. Assets will be projected based on an assumed return which for these purposes is also assumed to be the plan's actuarial valuation rate, 8%.

Therefore, under this theoretical world, the market value return on assets is equal to the assumed return. However, the source of return will be varied between income and appreciation. Further, let's assume an initial value of $100,000. If this fund is compared to the regulatory average value, the following relationships develop.

Regulatory Average Value as a Percent of Fair Market Value

Annual Cash Flow	Year	Return Attributable to Income* 50%	100%	150%**
$ 0	5	93.1%	100%	106.9%
	10	93.1	100	106.9
	15	93.1	100	106.9
10,000	5	93.8	100	106.2
	10	93.5	100	106.5
	15	93.4	100	106.6
(10,000)	5	91.5	100	108.5
	10	91.1	100	108.9
	15	90.1	100	109.9

*The remainder of the return is attributable to appreciation (or depreciation).
**e.g., a high coupon bond purchased at a premium.

As can be seen, if appreciation provides a portion of the total expected return, the 50% column, the regulatory average value method systematically understates the value of assets with such understatement increasing over time. This understatement occurs due to the delayed recognition of gains. Similarly, when depreciation occurs, the regulatory average value method overstates the value of assets with such overstatement increasing over time.

However, perhaps more interesting is the effect on actuarial gains and losses. Under immediate gain funding methods the amortization of these gains or losses directly affects plan funding requirements. Under spread gain funding methods, the gain or loss is reflected in adjustments to the normal cost.

Actuarial Asset Gain (Loss)

Annual Cash Flow	Return Attributable to Income 50%	100%	150%
$ 0	$ 0	$0	$ 0
10,000	(800)	0	800
(10,000)	800	0	(800)

Under the typical scenario, a portion of the return is generated by appreciation as illustrated in the first column (i.e., total asset returns typically exceed interest and dividends). The regulatory average value method systematically generates asset losses for plans with positive cashflows, the typical situation in less mature plans, and systematically generates asset gains for plans with negative cash flows. These biases occur because the appreciation on cashflow is not fully recognized until the expiration of the averaging period. Conversely, the reduction in return produced by a negative cash flow is recognized on a delayed basis thus generating gains.

Similarly, the opposite occurs where income exceeds the expected return and the recognition of depreciation is delayed. Positive cashflow produces actuarial gains while negative cashflows produce losses.

At this point, several undesirable characteristics of the regulatory average value method should be apparent.

- If appreciation (depreciation) in assets can be expected, average value systematically understates (overstates) the value of assets.
- Actuarial gains or losses will typically occur when interest and dividend income do not equal the assumed return even if the total return equals the assumed return (see equation 8).
- Investment strategies can be adopted which could influence the actuarial value of assets without any beneficial impact on return.
- In many situations the actuarial value of assets does not move towards the market value.

However, from year x to year x+1 this method does produce predictable results. As equation (7.2) indicates, the greatest variable is 1/n of the realized and unrealized gains and losses for most recent year. While interest and dividends may deviate somewhat from expectations, they can usually be fairly accurately estimated.

A simple modification to the regulatory average value method can be made which eliminates many of the above undesirable characteristics. The modification requires immediate recognition of the appreciation (depreciation) expected from assets instead of only immediately recognizing only interest and dividends. This assumes that some estimate of expected appreciation can be made.

With respect to fixed income investments being held to maturity, this is akin to using amortized basis running from initial cost at purchase to par value. Prior to 1987, such an approach was explicitly permitted by the Internal Revenue Code §412(c)(2)(B), but is not now available to non-multiemployer plans.

If fixed income investments are not held to maturity, it may be appropriate to anticipate a gain, or loss, attributable to movement of interest rates. This requires a degree of subjectivity in the selection of interest rates. Even greater subjectivity is required in the anticipation of appreciation of equity investments. However, this subjectivity should not be a deterrent to making an appropriate assumption. In fact, an assumption regarding investment return from all sources is already required in the valuation process. Furthermore, such an assumption for single employer plans is required under IRC §412 to be made on an explicit basis. Therefore, the most straightforward and consistent approach to immediately recognizing expected appreciation is to assume that the expected return on assets will equal the assumed valuation interest rate. Referring back to equation (8), this method creates an adjusted G_t^1 equal to

(9) $G_t^1 = R_t + U_t + I_t - (iA_x + i_t)$, and the actuarial gain becomes:

(10) $AG_x = \frac{1}{n}\sum_{x=0}^{n-1} G^1_{x-k}$

Use of such a method has received IRS approval under Revenue Procedure 98-10 and is equivalent to the basic market related value described in Statement of Financial Accounting Standards No. 87. If the assumed return is reasonable:

- The average value will reflect an underlying value that is a non-stationary.
- Interest and dividends are no longer differentiated from other investment performance.
- Over time, the actuarial value of assets will move towards the market value of assets.
- The source of return (income vs. appreciation) will not affect the actuarial value.
- Actuarial gains and losses will disappear if the actual return consistently equals the assumed interest rate.
- Investment strategies that affect the actuarial value of assets cannot be implemented.
- Predictability is further enhanced since an estimate of interest and dividends is no longer necessary.

Based on the model developed, the modification to immediately recognize expected return eliminates most of the shortcomings of the IRS average value method.

However, it is unclear how either method will respond to normal investment fluctuations. It is also necessary to model normal investment fluctuations to determine whether the fourth and sixth evaluation criterion are satisfied, i.e., in any given year does the actuarial value move towards market value and does the valuation method helps to minimize cumulative gains and losses.

The following table illustrates investment returns from 1970 through 1995 based on asset allocation of 50% in the S & P 500 and 50% in long-term U.S. Government Bonds. These returns have been derived from dividend, yield and price information contained in the Society of Actuaries "Statistics for Employee Benefits Actuaries," April, 1997. The table splits returns between appreciation and income. Appreciation was determined based on an arithmetic average of the percentage change in price per share of the S & P 500 and the percentage change in the bond price index. Similarly, income was determined based on an arithmetic average of the S & P dividend yield and the bond yield.

Historical Returns of a Balanced Bond/Equity Portfolio

Year	Return from Appreciation	Return from Income	Total Return
1970	3.66%	5.07%	8.73%
1971	8.50	4.62	13.12
1972	7.74	4.38	12.12
1973	(17.41)	5.30	(12.11)
1974	(18.71)	6.71	(12.00)
1975	15.29	6.22	21.51
1976	14.01	5.91	19.92
1977	(9.53)	6.35	(3.18)
1978	(4.57)	6.95	2.38
1979	(1.28)	7.31	6.02
1980	7.35	8.01	15.36
1981	(10.87)	9.62	(1.25)
1982	22.24	9.08	31.32
1983	2.78	7.86	10.64
1984	2.07	8.65	10.72
1985	24.71	7.45	32.16
1986	17.97	5.72	23.70
1987	(6.50)	6.13	(0.37)
1988	5.93	6.33	12.26
1989	19.44	5.90	25.34
1990	(4.84)	6.22	1.37
1991	18.27	5.63	23.89
1992	2.28	5.32	7.60
1993	10.57	4.73	15.30
1994	(9.44)	5.15	(4.29)

Applying these returns to an initial investment of $100,000 with an annual net capital inflow of $10,000, the IRS regulatory average value method can be compared to the modification described above; i.e., immediately recognizing an expected return based on an assumed rate of interest. For this purpose two assumed rates will be evaluated. 8% will be used as reflective of more typically assumed valuation interest rates, and 10% will be used as reflective of the actual return from 1970 to 1995 of this model. The actual annual compound return during the period for this 50%/50% portfolio is 10.65%. Note that no provision has been made for investment expenses which can have material effect on returns actually realized.

		Immediate Recognition of Income		Immediate Recognition of 8% Return		Immediate Recognition of 10% Return	
Year	Beginning Market Value	Beginning Actuarial Value	Actuarial Gain	Beginning Actuarial Value	Actuarial Gain	Beginning Actuarial Value	Actuarial Gain
1970	100,000	100,000	(2,195)	100,000	146	100,000	(254)
1971	118,732	115,805	(1,031)	118,146	1,371	119,746	466
1972	144,305	134,039	579	138,969	2,645	142,187	1,120
1973	171,793	155,341	(4,316)	162,731	(4,119)	167,526	(6,390)
1974	160,995	173,452	(10,089)	181,631	(10,887)	187,888	(14,011)
1975	151,681	187,239	(8,668)	195,274	(7,634)	202,666	(11,286)
1976	194,303	203,551	(4,489)	213,262	(4,528)	221,647	(8,696)
1977	243,016	225,345	(9,148)	235,795	(11,119)	245,115	(15,797)
1978	245,292	244,225	(5,308)	253,540	(7,245)	263,830	(12,396)
1979	261,131	268,456	145	276,578	(1,755)	287,817	(7,385)
1980	286,863	300,077	1,103	306,950	(1,253)	319,214	(7,427)
1981	340,928	335,186	(4,766)	340,253	(11,880)	353,708	(18,664)
1982	346,663	367,235	11,422	365,592	9,304	380,414	1,891
1983	465,244	418,035	17,268	414,143	15,470	430,346	7,296
1984	524,760	478,746	24,085	472,744	20,436	490,677	11,344
1985	591,017	551,131	41,947	541,000	45,898	561,088	35,713
1986	791,116	647,168	71,344	640,178	79,450	662,910	67,880
1987	988,604	780,285	47,757	780,842	50,356	807,080	36,896
1988	994,936	900,465	49,720	903,665	57,020	934,685	41,306
1989	1,126,933	1,032,223	84,348	1,042,978	93,760	1,079,459	75,394
1990	1,422,508	1,209,149	49,117	1,230,176	48,621	1,272,799	27,048
1991	1,452,047	1,364,998	54,536	1,387,211	68,564	1,437,126	43,459
1992	1,809,013	1,538,733	76,268	1,576,751	84,048	1,634,298	55,133
1993	1,956,438	1,748,099	85,346	1,796,940	105,228	1,862,861	72,042
1994	2,265,798	1,983,293	4,228	2,055,923	12,483	2,131,189	(25,530)
1995	2,178,685	2,156,185		2,242,880		2,328,777	

Examining the market values at the beginning of 1974, 1975 indicates that they were lower than the market value at the beginning of the preceding year. In each case, the actuarial value was greater than the preceding years market value and increased during the year. This occurred under each of the asset valuation methods. This illustrates that the average value method, whether or not it is modified, does not always move toward the current market value. Of course, reviewing the mathematical derivation of the methods helps to illuminate why this occurs. After a gain or loss has been established, it is recognized gradually irrespective of subsequent asset performance. Inevitably, experience will occur when the cumulative gains (or losses) being recognized in a year are greater in value than the current loss (or gain) being generated. When this occurs, this actuarial value and the market value of assets should be expected to cross paths and continue to diverge.

Therefore, neither of these average value methods universally satisfies the fourth enumerated criterium. The following table illustrates that valuation methods ability to reduce cumulative gains and losses:

Actuarial Value	Assumed Return	Cumulative Actuarial Gains (Losses) 1970 to 1995*
Market value	8%	$456,356
	10%	115,273
Regulatory average value	8%	536,702
	10%	237,457
Modified average value	8%	570,183
	10%	199,058

*Average values have been adjusted to reflect gains not fully recognized as of the beginning of 1995.

As these results illustrate, over the 25 year period, use of straight market value would have produced lower gains and losses than either of the average value methods. Thus, the average value methods being examined do not effectively smooth out gains and losses. They affect the incidence of gains and losses but in this situation were not able to effectively counterbalance asset gains and losses in different years to reduce overall actuarial gains and losses.

Referring back to the table of actuarial values, the preponderance of the actuarial losses occurring in the early years under each of the methods is alarming. Similarly, the actuarial value of assets much more frequently exceeded the market value in these same years. These losses are reflective of the smoothing methods' gradual recognition of losses. This pattern suggests that the annual "true" appreciation is not stable during the period in question, 1970 through 1995. If this is the case, and presuming that the true rate of appreciation cannot be reasonably estimated on a prospective basis at any point in time, it is difficult to devise an average value asset valuation method which will exhibit the stated desirable characteristics in actual application.

Comparison of the actuarial values and gains or losses does not indicate the clear superiority of either method. This is primarily because historical returns are extremely erratic. Therefore, while it may be theoretically appealing to devise an average value asset valuation method which does not produce actuarial gains or losses under certain conditions, in reality it is highly unlikely that those conditions will ever be met. However, if investment decisions are influenced by the characteristics of the actuarial asset valuation method, such a method is clearly inappropriate.

At this point, it may be helpful to summarize the characteristics of a desirable asset valuation method as they apply to alternative average value methods. This summary is presented in the following table:

Desirable Characteristics	Average Value Method: Immediate Recognition of Dividends and Interest	Average Value Method: Immediate Recognition of Expected Return
Does not consistently overstate or understate asset value	No, if investments anticipate any appreciation	Only if the assumed return is a reasonable long term estimate of actual returns
Produces values which will converge on market value	No	Only if the assumed return can accurately reflect long term market returns
Is not influenced by choice of investments	No	Yes
Actuarial value consistently moves towards market value	No	No
Results are predictable	Recognition of past gains and losses is predictable but dividends and interest must be reasonably estimated	Both the expected return and the recognition of past gains and losses is predictable
Reduces overall net gains (losses)	No	No

As this table indicates, the use of an expected return produces more desirable characteristics for an average value asset valuation method as compared to the regulatory average value method, but does not produce a clearly superior method. This failure occurs primarily because historically asset returns have not fluctuated around any level on a consistent long term basis. In other words, the asset returns have not exhibited a stable mean. As a result, any average value method will be unable to exhibit the aforementioned desirable characteristics unless an explicit mean return can be reliably predicted on a prospective basis.

This suggests that it may be more appropriate to utilize other asset valuation methods which do not rely on averaging to produce the actuarial value of assets. Such methods might borrow from smoothing techniques used in other areas of actuarial science, particularly methods which focus on the end points of series of observations. The addition of contingency reserve techniques could also be used to minimize asset gains and losses. While it is beyond the scope of this paper to develop such methods, the evaluation criteria developed can be used to assess their efficacy.